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Determination of the Distance and Velocity of an Acoustic Source From Bearing and Frequency Measurements

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PREFACE

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13. ABSTRACT (Maximum 200 words)

In this report, a technique is developed that uses frequency and bearing measurements to determine the distance and velocity of a steady sinusoidal acoustic source moving with constant velocity relative to a receiver (which may or may not be in motion).

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DETERMINATION OF THE DISTANCE AND VELOCITY OF AN ACOUSTIC SOURCE FROM BEARING AND FREQUENCY MEASUREMENTS

INTRODUCTION

A technique that uses bearing and frequency measurements to determine the distance between an observer and a sinusoidal acoustic source moving at constant velocity relative to the observer is developed in this report.

THEORETICAL ANALYSIS

Consider a sinusoidal acoustic source with period T_e and fixed relative velocity $\vec{v}_R$ (having a nonzero transverse component) with respect to an observer with known velocity $\vec{v}$. Let $\vec{r}$ be the position vector of the source from the observer. We wish to determine the position vector $\vec{r}$ and the relative velocity $\vec{v}_R$ of the source relative to the observer from bearing θ and frequency f_e measurements. The absolute velocity $\vec{v}_s$ of the source is also determined.

Since we are interested in obtaining kinematic quantities subject to the constraint that the acceleration is zero, let us examine this aspect of the problem. For convenience, we use plane polar coordinates. For the position vector, we have

$$\vec{r}(\theta) = r\,\hat{r}(\theta) \quad , \tag{1}$$

where

$$\hat{r}(\theta) = \hat{i}\cos\theta + \hat{j}\sin\theta \tag{2}$$

and

$$\hat{\theta}(\theta) = -\hat{i}\sin\theta + \hat{j}\cos\theta \quad . \tag{3}$$

Differentiating the unit vectors with respect to θ, we have

$$\frac{\partial \hat{r}}{\partial \theta} = \hat{\theta} \tag{4a}$$

and

$$\frac{\partial \hat{\theta}}{\partial \theta} = -\hat{r} \quad . \tag{4b}$$

The time derivative of $\vec{r}$ is

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}(\theta)}{dt} = \dot{r}\,\hat{r} + r\,\dot{\theta}\,\hat{\theta} \quad . \tag{5}$$

Differentiating again, we have

$$\ddot{\vec{r}} = \left(\ddot{r} - r\,\dot{\theta}^2\right)\hat{r} + \left(r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}\right)\hat{\theta} \quad . \tag{6}$$

Since by construction the relative velocity between the source and the observer is fixed, the acceleration must be identically zero. Consequently, all linearly independent components of the acceleration must be zero. Thus, we obtain the following expressions:

$$\ddot{r} = r\,\dot{\theta}^2 \tag{7}$$

and

$$r = -\,2\,\dot{r}\,\dot{\theta}\,/\,\ddot{\theta} \quad . \tag{8}$$

The quantities $\dot{\theta}$ and $\ddot{\theta}$ are obtained from measurements of θ. The quantities $\dot{r}$ and $\ddot{r}$ are not measurable but are related to changes in the observed acoustic frequency of the source. Therefore, we seek the relationship involving $\dot{r}$, $\ddot{r}$, and the measured frequency and its time derivatives.

At time t_0, let the sinusoidal source emit a "pulse" and let $\vec{r} = \vec{r}_1$ so that the magnitude of $\vec{r}$ is r_1. After one period T_e, the source again emits a pulse at time t_0+T_e. Let $\vec{r} = \vec{r}_2$ so that the magnitude of $\vec{r}$ is r_2. Suppose that the first pulse is heard by the observer at time t_1 and the second pulse at time t_2. Denoting the speed of sound as c, we have

$$t_1 = t_0 + \frac{r_1}{c} \tag{9}$$

and

$$t_2 = t_0 + T_e + \frac{r_2}{c} \quad . \tag{10}$$

The period T_r of the acoustic signal as seen by the observer is

$$T_r = t_2 - t_1 \quad . \tag{11}$$

Substitution of the expressions for t_1 and t_2 in equations (9) and (10) into equation (11) yields

$$T_r = T_e + \frac{r_2 - r_1}{c} \quad . \tag{12}$$

The ratio of T_r to T_e is

$$\frac{T_r}{T_e} = 1 + \frac{r_2 - r_1}{cT_e} \quad . \tag{13}$$

In equation (13), we identify an expression for the radial velocity $\dot{r}$ as follows:

$$\dot{r} = \frac{r_2 - r_1}{T_e} \quad . \tag{14}$$

Thus, we obtain

$$\frac{T_r}{T_e} = 1 + \frac{\dot{r}}{c} \quad . \tag{15}$$

Letting $f_e = 1/T_e$ and $f_r = 1/T_r$, we have

$$f_e = f_r\left(1 + \frac{\dot{r}}{c}\right) \quad . \tag{16}$$

Differentiating equation (16) with respect to time, we obtain

$$\frac{df_e}{dt} = \frac{\partial f_r}{\partial t} + \frac{\dot{r}}{c}\frac{\partial f_r}{\partial t} + \frac{f_r}{c}\frac{\partial \dot{r}}{\partial t} \quad . \tag{17}$$

Since f_e is constant by hypothesis, some rearrangement yields

$$0 = \frac{\partial f_r}{\partial t} + \frac{\dot{r}}{c}\frac{\partial f_r}{\partial t} + \frac{f_r}{c}\frac{\partial \dot{r}}{\partial t} \tag{18}$$

and

$$\frac{\partial f_r}{\partial t} = -\frac{\dot{r}}{c}\frac{\partial f_r}{\partial t} - \frac{f_r}{c}\ddot{r} \quad . \tag{19}$$

Substituting in the expressions obtained above for $\dot{r}$ (eq. 8) and $\ddot{r}$ (eq. 7), we have

$$\frac{\partial f_r}{\partial t} = -\left(-\frac{r\ddot{\theta}}{2\dot{\theta}}\right)\frac{1}{c}\frac{\partial f_r}{\partial t} - \frac{f_r}{c}\left(r\,\dot{\theta}^2\right) \quad . \tag{20}$$

We solve this to obtain an expression for r as follows:

$$\frac{\partial f_r}{\partial t} = r\,\frac{\ddot{\theta}}{2\dot{\theta}}\,\frac{1}{c}\,\frac{\partial f_r}{\partial t} - r\,\frac{f_r}{c}\,\dot{\theta}^2 \quad , \tag{21}$$

$$\frac{\partial f_r}{\partial t} = \frac{r}{c}\left[\frac{\ddot{\theta}}{2\dot{\theta}}\,\frac{\partial f_r}{\partial t} - f_r\,\dot{\theta}^2\right] \quad , \tag{22}$$

$$r = c\,\frac{\partial f_r}{\partial t}\left[\frac{\ddot{\theta}}{2\dot{\theta}}\,\frac{\partial f_r}{\partial t} - f_r\,\dot{\theta}^2\right]^{-1} \quad . \tag{23}$$

Note that, in this equation for r, all the independent parameters are known or may be determined from measurements. The sound speed c is generally known *a priori*. The quantities θ and f_r are measurable, and $\dot{\theta}$, $\ddot{\theta}$, and $\partial f_r/\partial t$ can be determined from these measurements.

Since θ may be measured directly and r may be calculated from measured quantities, we have the position vector $\vec{r}$ of the source with respect to the observer. If we determine the position vector at two instants t_2 and t_1, we may calculate the relative velocity $\overrightarrow{v_R}$ of the source as follows:

$$\overrightarrow{v_R} = \frac{d\vec{r}}{dt} = \frac{\vec{r_2}(\theta_2) - \vec{r_1}(\theta_1)}{t_2 - t_1} \quad . \tag{24}$$

Recall that the relative velocity $\overrightarrow{v_R}$ of the source from the perspective of the observer is

$$\overrightarrow{v_R} = \vec{v_s} - \vec{v} \quad . \tag{25}$$

Therefore, if the relative velocity $\overrightarrow{v_R}$ and the absolute velocity $\vec{v}$ of the observer are known, then the absolute velocity of the source may be obtained:

$$\vec{v_s} = \overrightarrow{v_R} + \vec{v} \quad . \tag{26}$$

CONCLUSIONS

We have shown how to calculate the distance, the relative velocity, and the absolute velocity of an acoustic source having a fixed velocity relative to an observer from measurements of its bearing and frequency.

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